Tangling Clustering in Stratified Turbulent Flows

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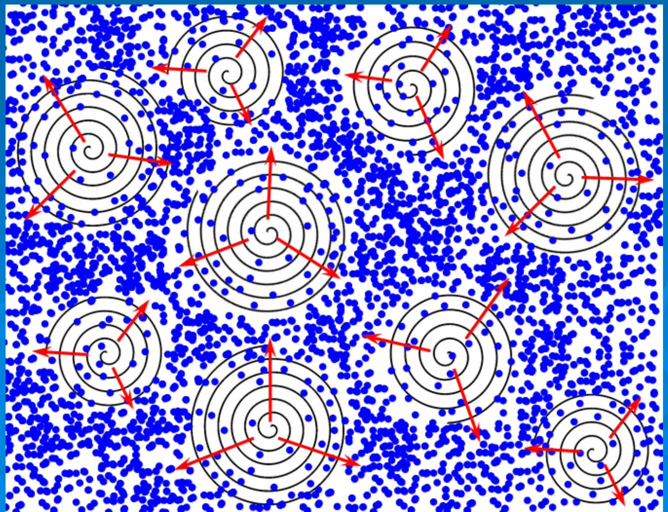
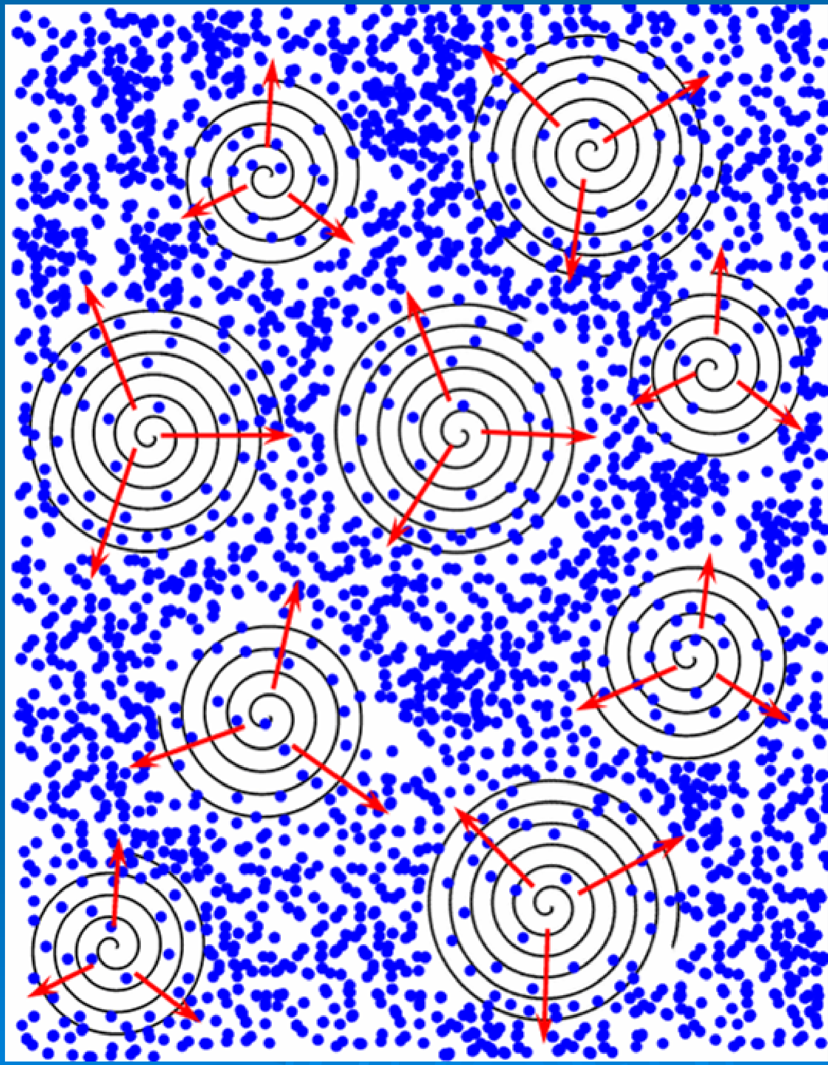


Inertial Clustering of Small Solid Particles

- Inertia causes particles inside the turbulent eddies to drift out to the boundary regions between the eddies (i.e. regions with low vorticity or high strain rate and maximum of fluid pressure).
- This mechanism acts in a wide range of scales of turbulence.
- Scale-dependent turbulent diffusion causes relaxation of particle clusters.
- In small scales

$$D_T(\ell) \to D_m$$

- Thus, clusters of particles are localized in small scales.



Experimental Study of Inertial Clustering

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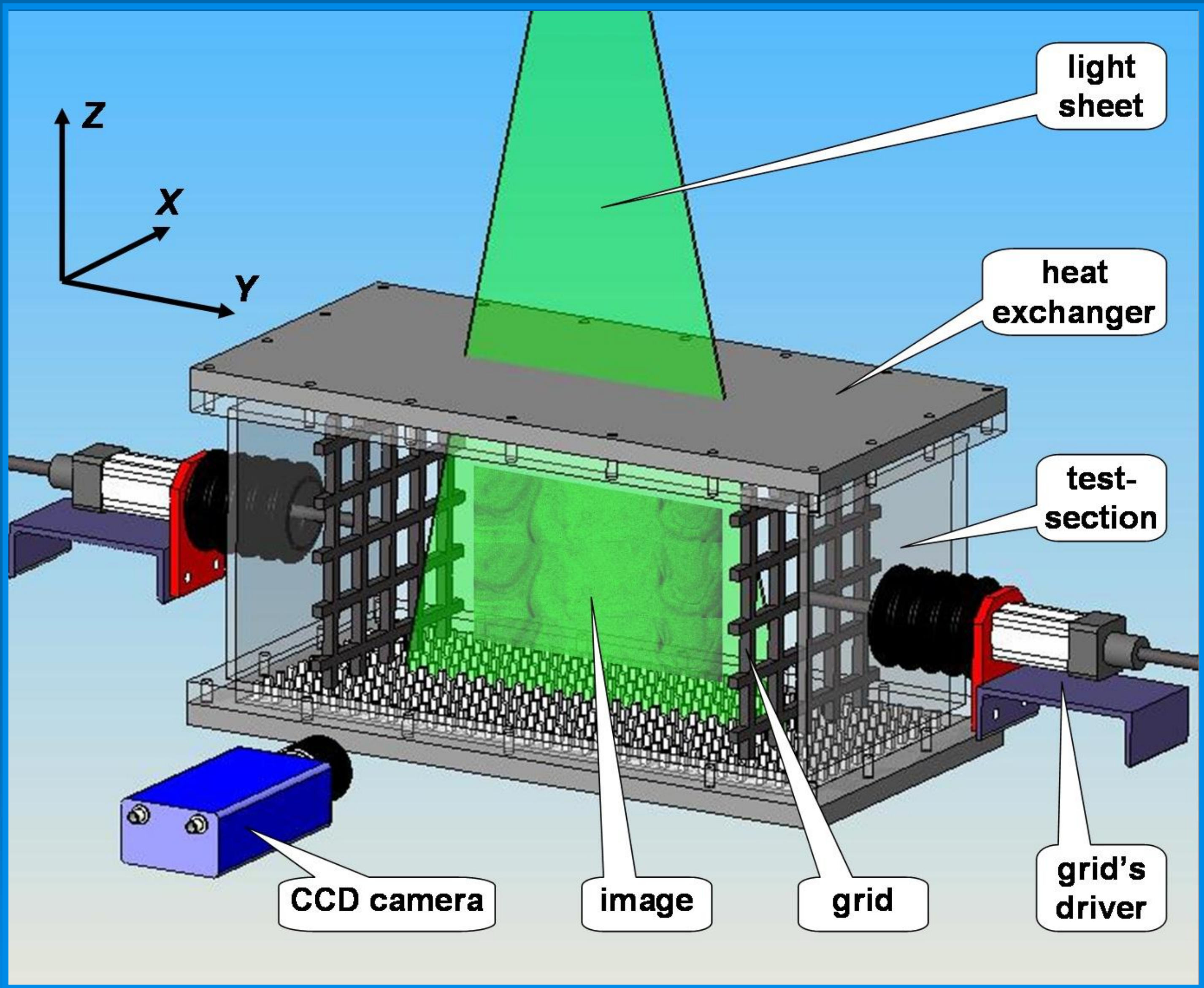
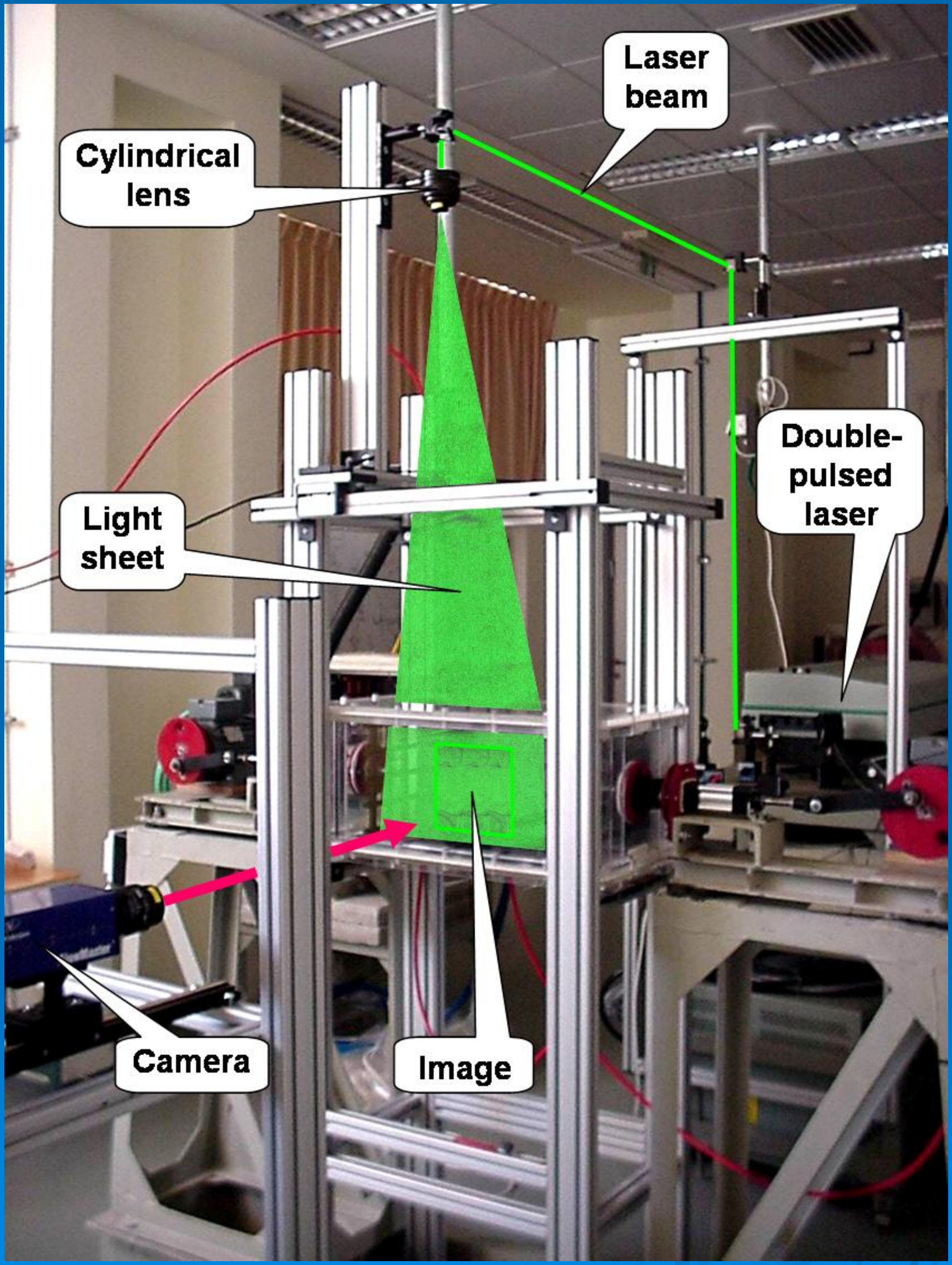
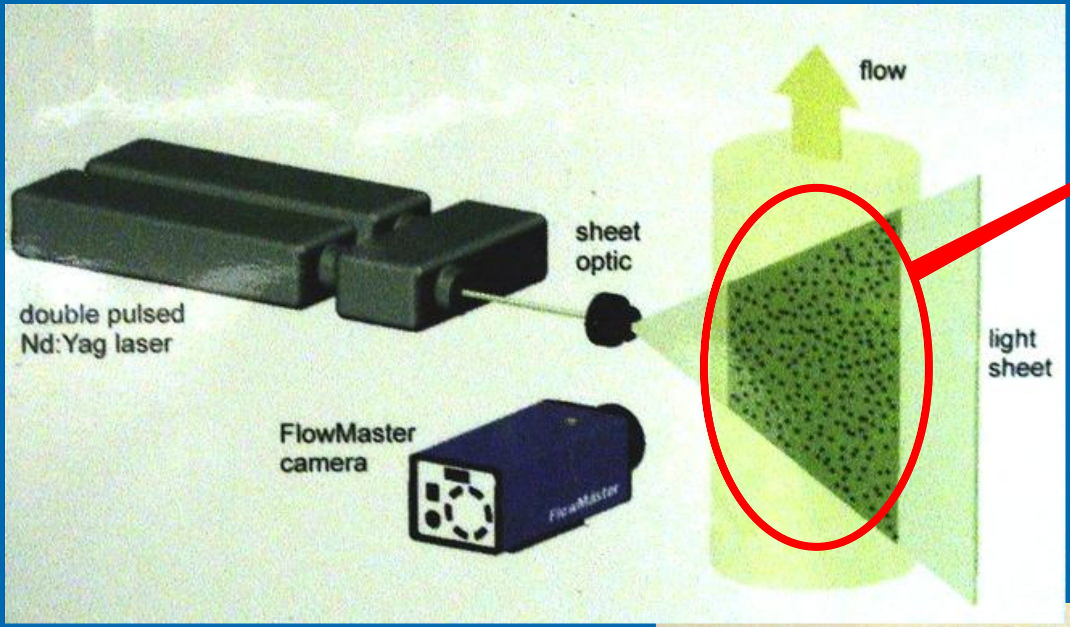
Experimental Set–up for Tangling Clustering





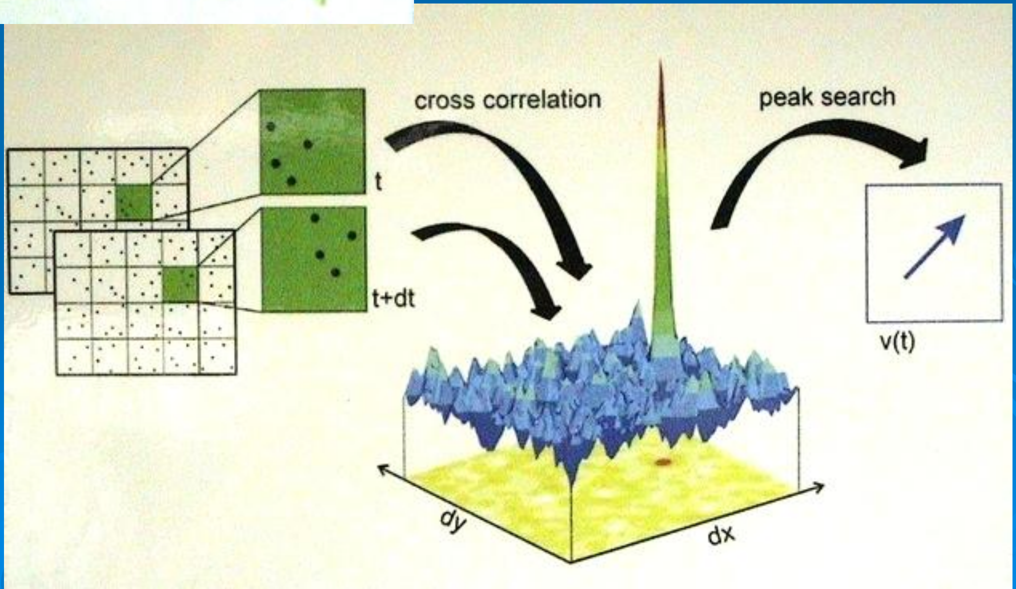
Experimental set - up: oscillating grids turbulence generator and particle image velocimetry system

Particle Image Velocimetry System



Raw image of the incense smoke tracer particles in oscillating grids turbulence

Particle Image Velocimetry Data Processing



Parameters of Turbulence and Solid Particles in Experimental Study of Tangling Clustering in Air

$u_0 = \sqrt{\langle \mathbf{u}^2 \rangle} = 12\,\mathrm{cm/s}$ **is the r.m.s. velocity;**

$\ell_0 = 3.2\,\mathrm{cm}$ **is the integral (maximum) scale of turbulence;**

$\mathrm{Re} = u_0\, \ell_0 / \nu = 250$ **is the Reynolds numbers;**

$\ell_\eta = \ell_0 / \mathrm{Re}^{3/4} = 510\,\mu\mathrm{m}$ **is the Kolmogorov length scale;**

$\tau_\eta = \tau_0 / \mathrm{Re}^{1/2} = 1.7 \times 10^{-2}\,\mathrm{s}$ **is the Kolmogorov time scale;**

$d_p = 10\,\mu\mathrm{m}$ **is the particle diameter;**

$\tau_s = 10^{-3}\,\mathrm{s}$ **is the Stokes time for the particles;**

$\mathrm{St} = \tau_s / \tau_\eta = 6 \times 10^{-2}$ **is the Stokes number for the particles;**

$\mathrm{Pe} = u_0\, \ell_0 / D_m = 3 \times 10^9$ **is the Peclet number for the particles;**

$D_m = 1.4 \times 10^{-8}\,\mathrm{cm}^2/\mathrm{s}$ **is the coefficient of molecular diffusion;**

Experimental Study of Tangling Clustering

- **Two-point correlation function of particle number density:**

$$\Phi(t,\mathbf{R}) = \langle n'(t,\mathbf{x})\, n'(t,\mathbf{x}+\mathbf{R})\rangle = N^2\,[G(t,\mathbf{R})-1]$$

- **Radial distribution function can be estimated as follows:**

$$G(\mathbf{R}) \approx \frac{N^{(p)}_{\Delta S}/\Delta S}{N^{(p)}_{S}/S}$$

$$n = N + n', \quad N = \langle n \rangle$$

$N^{(p)}_{\Delta S}$ **is the number of particle pairs separated by a distance:** $R \pm \frac{1}{2}\Delta R$

ΔS **is the area of the annular domain located between:** $R \pm \frac{1}{2}\Delta R$

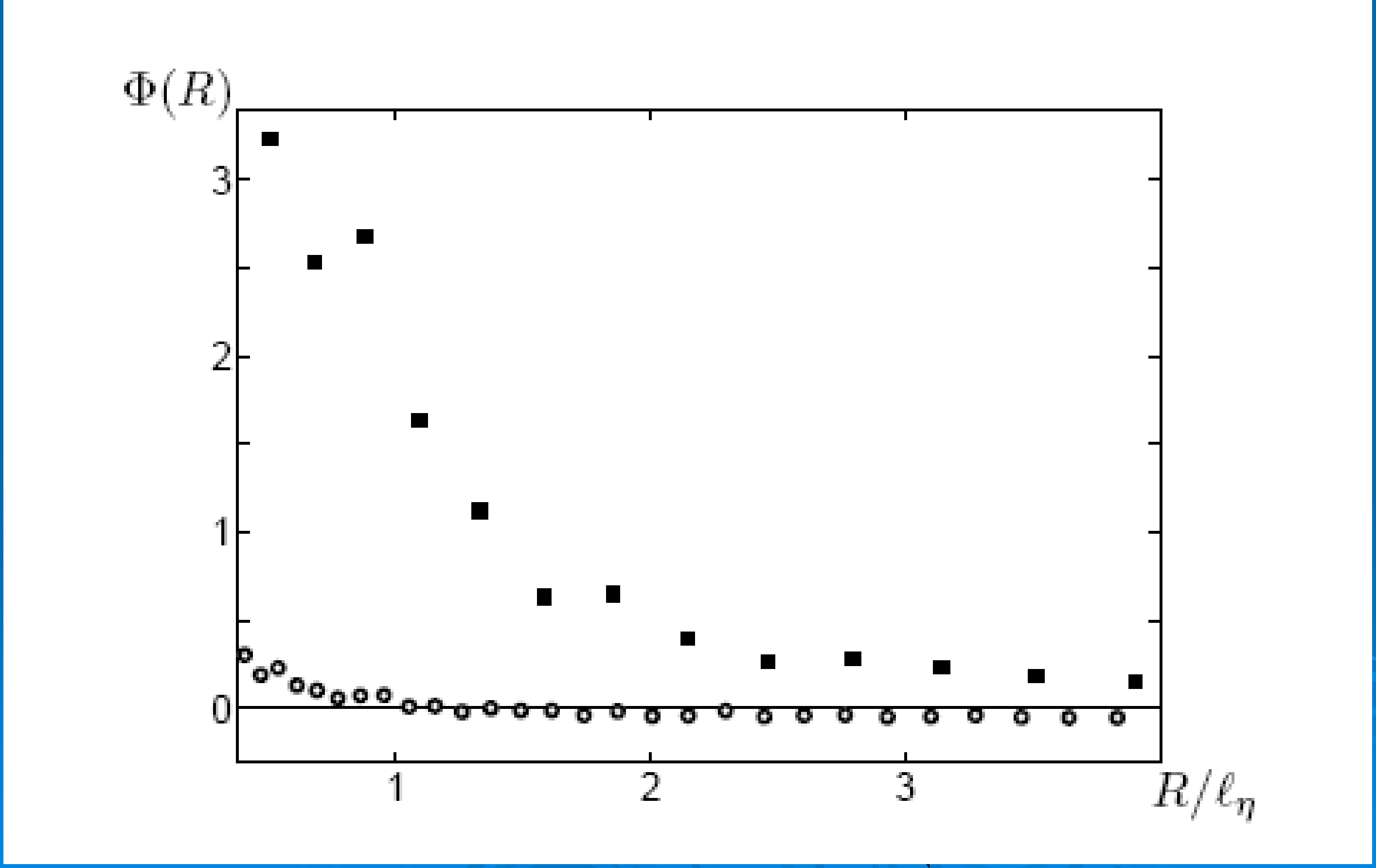
S **is the area of the part of the image with the radius:** $R_{\max} = 0.8\,\mathrm{cm}$

$N^{(p)}_{S} = \frac{1}{2}M(M-1)$ **is the total number of pairs in the area:** S

$M \sim 10^3$ **is the total number of particles in the area:** S

We perform the double averaging (i) over all particles in the image and (ii) over ensemble of 50 images.

Normalized second-order correlation function determined in our experiments for (i) inertial clustering (isothermal turbulence, circles) (ii) tangling clustering (non-isothermal turbulence, squares)



Particle Fluctuations in Turbulent Flow

- **Instantaneous particle number density:**

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\cdot(n\,\mathbf{v}) - D_m\,\Delta n = 0$$

- **Mean particle number density:** $n = N + n', \qquad N = \langle n \rangle$

$$\frac{\partial N}{\partial t} + \boldsymbol{\nabla}\cdot\langle n\,\mathbf{v}\rangle - D_m\,\Delta N = 0$$

$$\boldsymbol{\nabla} T \neq 0$$

$$\boldsymbol{\nabla} N \neq 0$$

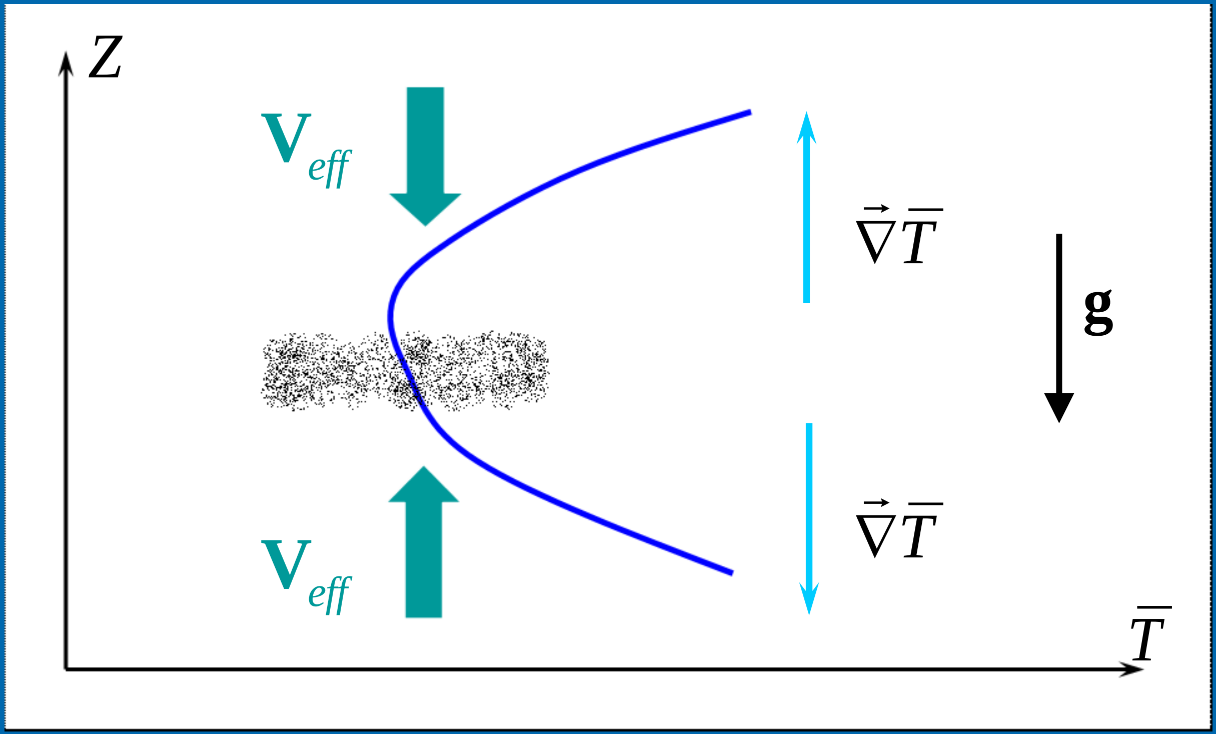
- **Fluctuations of particle number density:**

$$\frac{\partial n'}{\partial t} + \boldsymbol{\nabla}\cdot(n'\,\mathbf{v} - \langle n'\,\mathbf{v}\rangle) - D_m\,\Delta n' = -N\,(\boldsymbol{\nabla}\cdot\mathbf{v}) - (\mathbf{v}\cdot\boldsymbol{\nabla})N$$

- **Source of fluctuations:**

$$I_{n'} = -N\,(\boldsymbol{\nabla}\cdot\mathbf{v}) - (\mathbf{v}\cdot\boldsymbol{\nabla})N$$

Turbulent Thermal Diffusion



$$\mathbf{V}_{\rm eff} = -\langle \tau\, \mathbf{v}(\mathbf{x})\, \boldsymbol{\nabla}\cdot \mathbf{v}(\mathbf{x}) \rangle$$

$$\mathbf{V}_{eff} = -D_T \left(1 + \left(\frac{m_p}{m_\mu}\right) \left(\frac{\bar{T}}{T_*}\right) \frac{\ln(\mathrm{Re})}{\mathrm{Pe}} \right) \frac{\nabla \bar{T}}{\bar{T}}$$

Derivation of Effect of Turbulent Thermal Diffusion

- **All known approaches (including dimensional reasoning)**
- **Path integral approach (finite correlation time);**
- **The spectral tau approximation; Quasi-linear approach, etc.**

T. Elperin, N. Kleeorin and I. Rogachevskii

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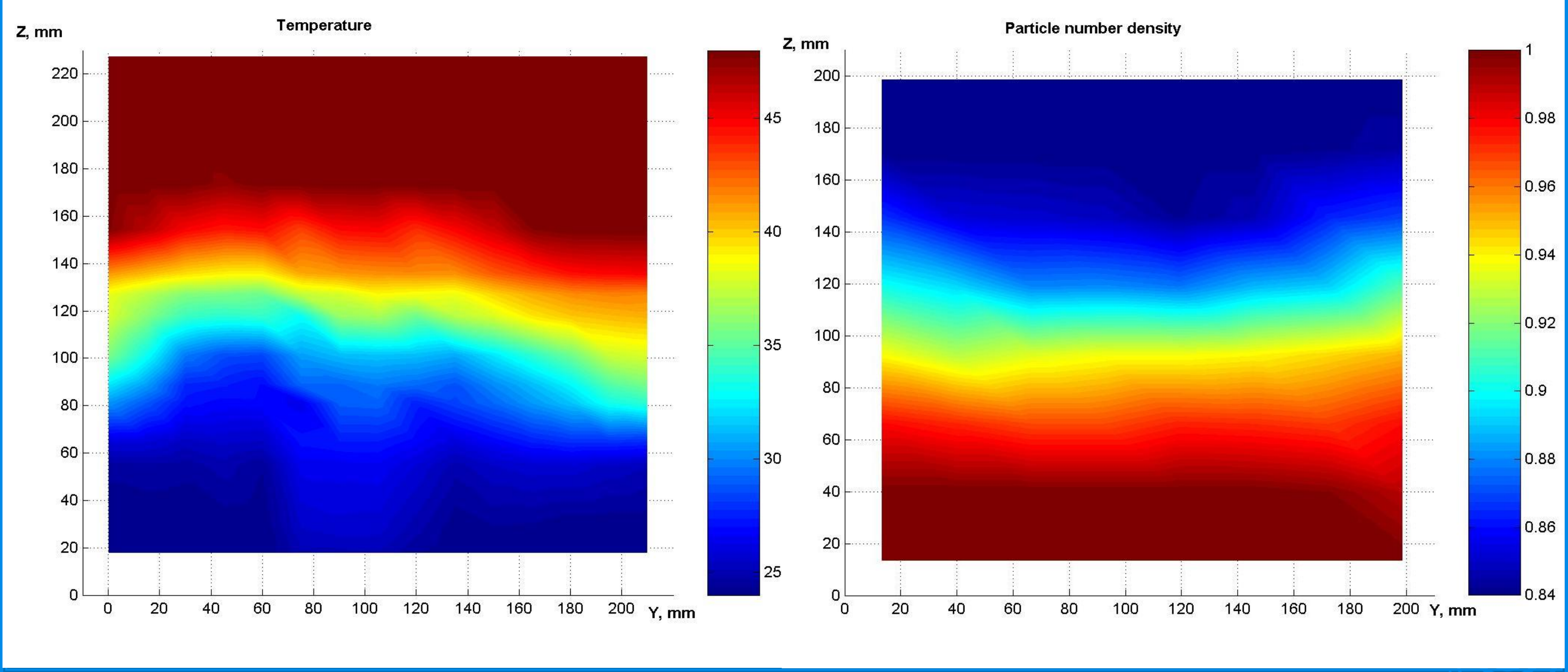
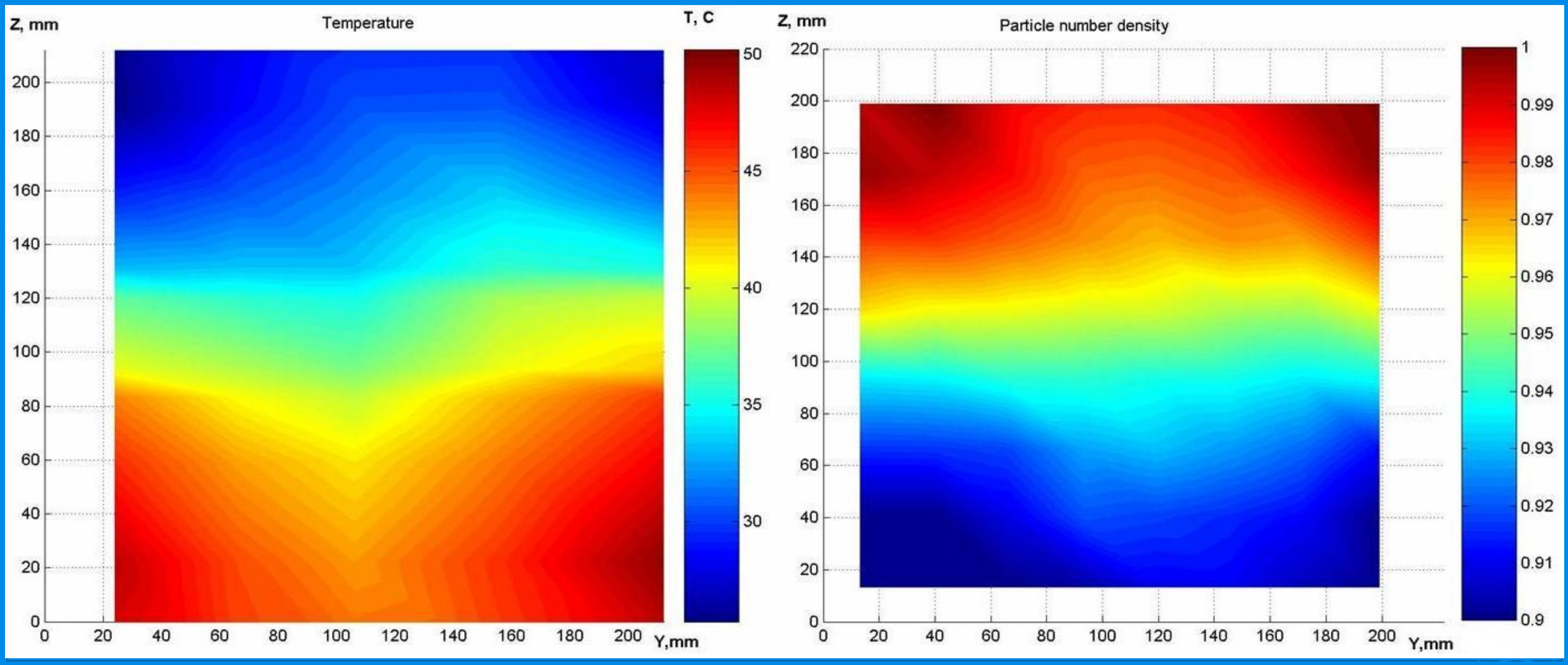
Temperature and Particle Number Density Fields. Stable Stratification



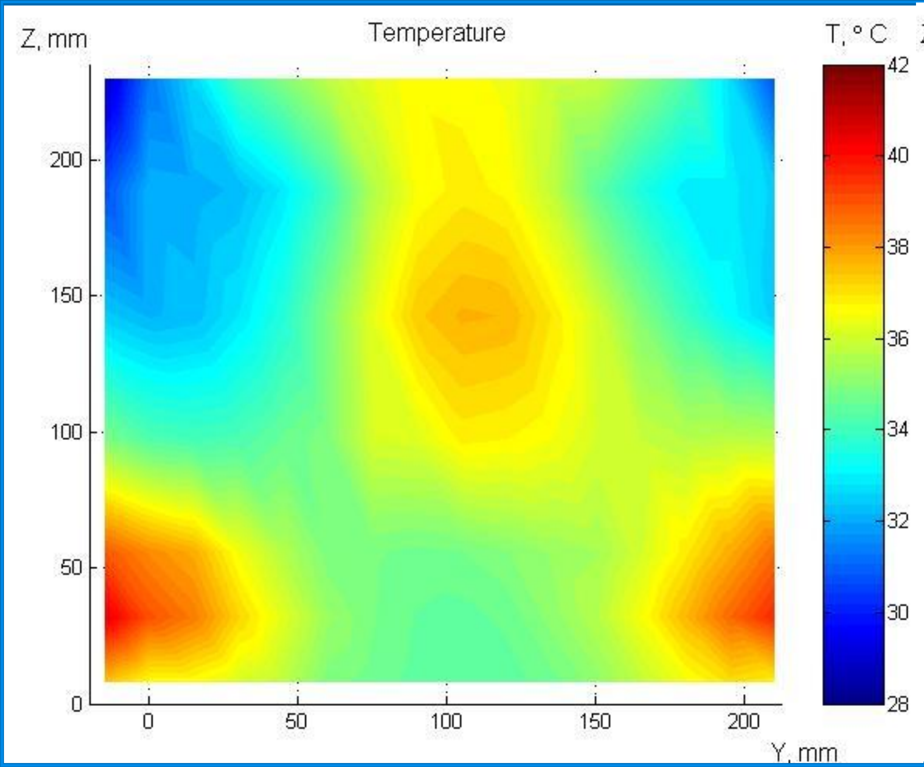
$T(y,z)$ $N(y,z)$

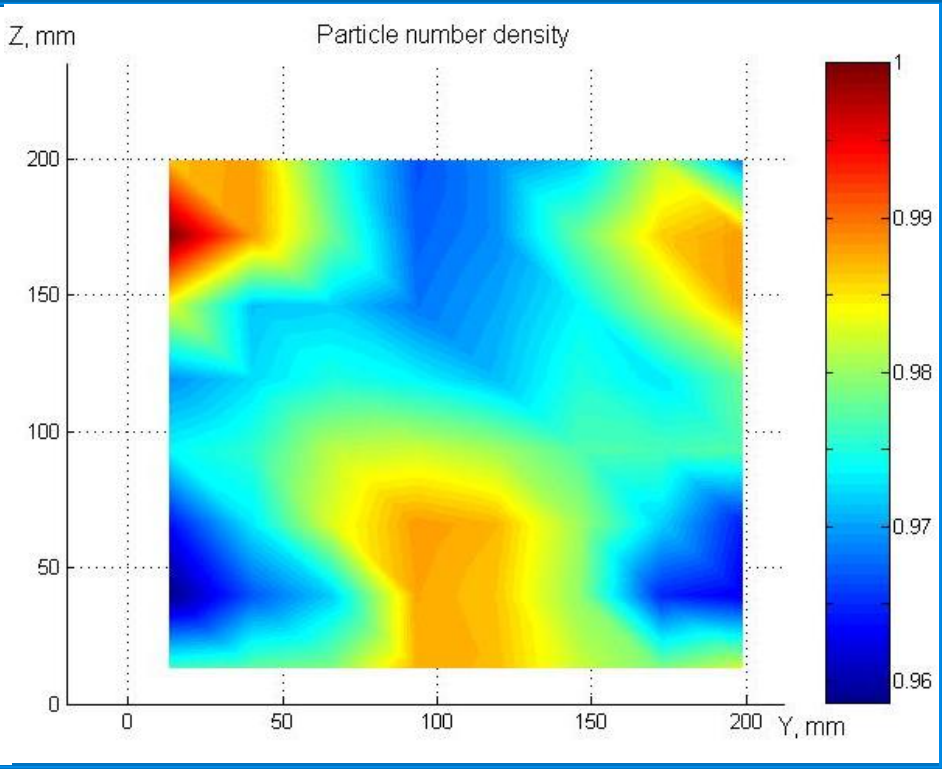
Temperature and Particle Number Density Fields. Unstable Stratification, f = 10.5 Hz



$$T(y,z) \qquad N(y,z)$$

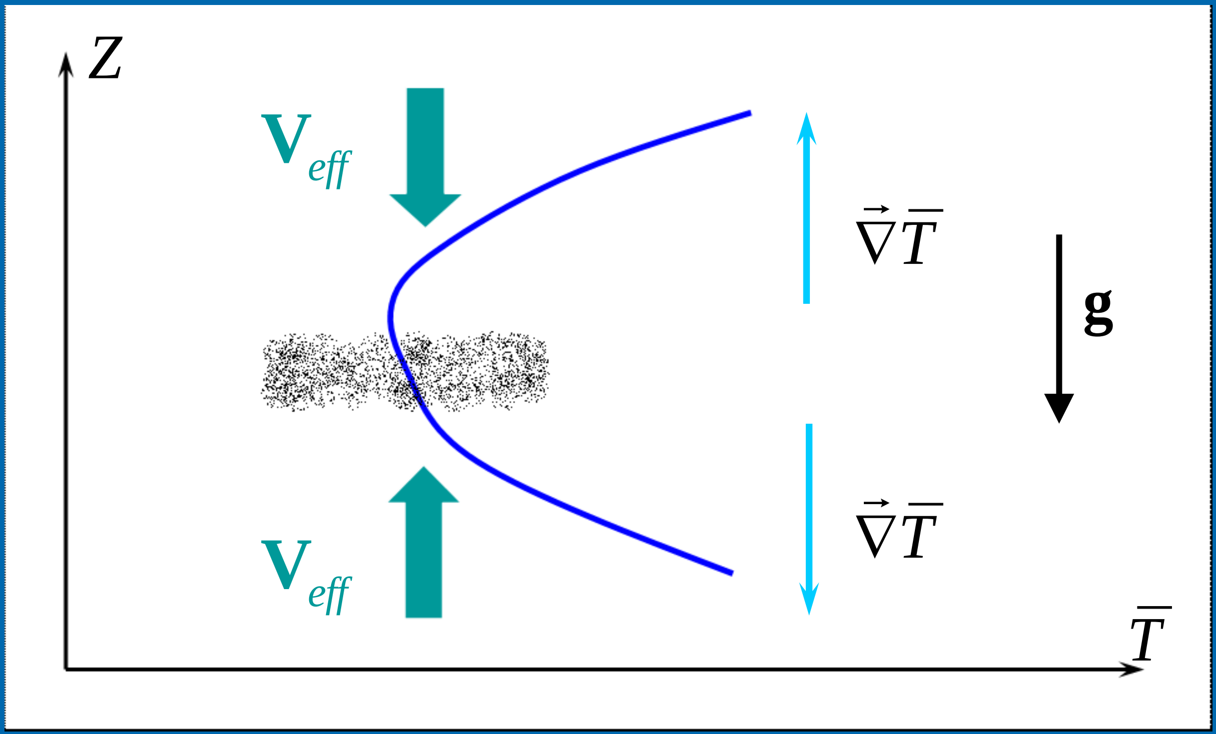
Temperature and Particle Number Density Fields. Unstable Stratification, f = 4.4 Hz





$T(y,z)$ $N(y,z)$

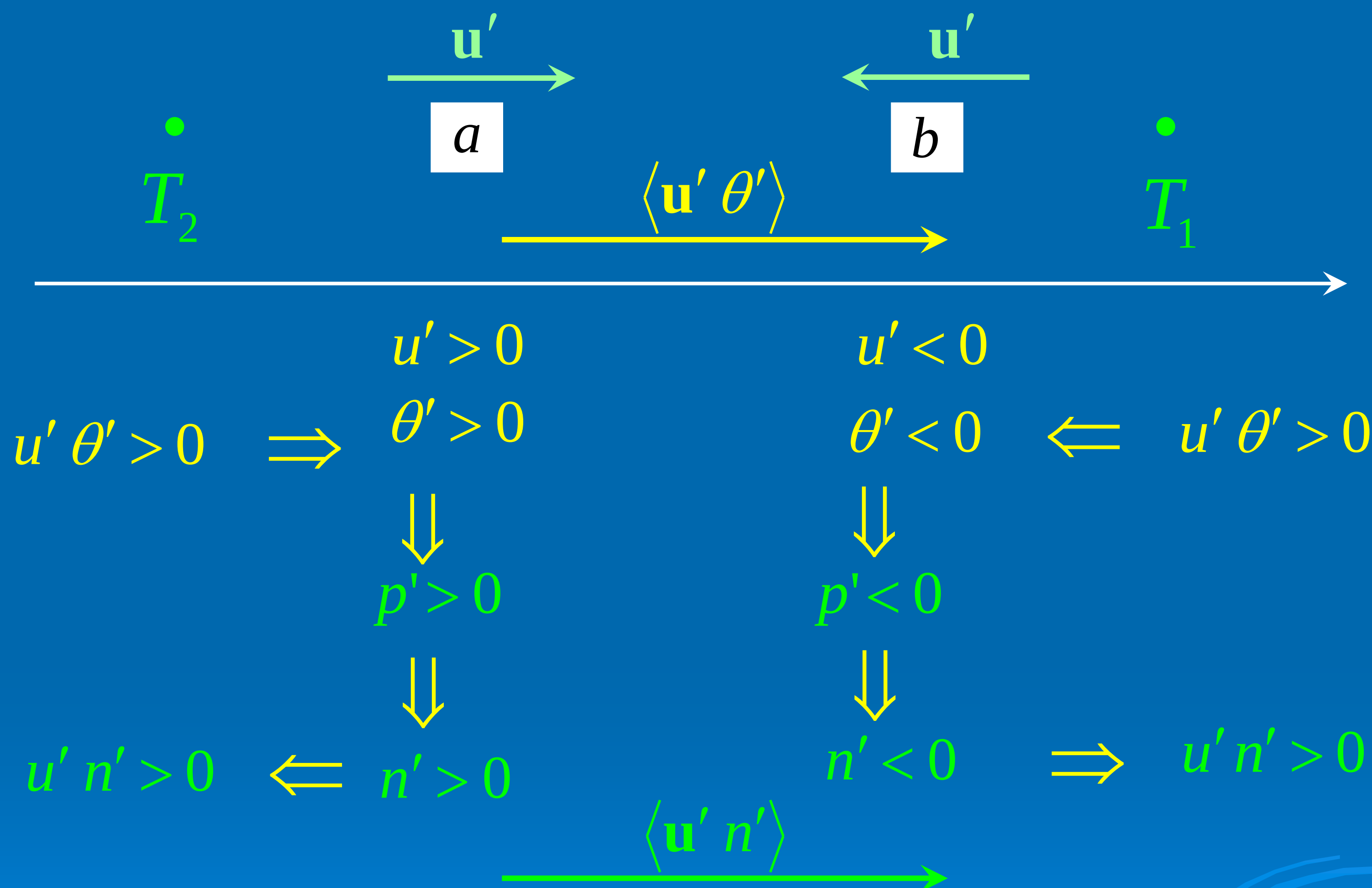
Turbulent Thermal Diffusion



$$\mathrm{V_{eff}} = -\langle \tau\, \mathbf{v}(\mathbf{x})\, \boldsymbol{\nabla}\cdot \mathbf{v}(\mathbf{x}) \rangle = -\frac{\tau_s}{\rho} \langle \tau\, \mathbf{v}(\mathbf{x})\, \nabla^2 p(\mathbf{x}) \rangle \approx -\frac{\tau_s P}{\rho T} \langle \tau\, \mathbf{v}(\mathbf{x})\, \nabla^2 \theta(\mathbf{x}) \rangle$$

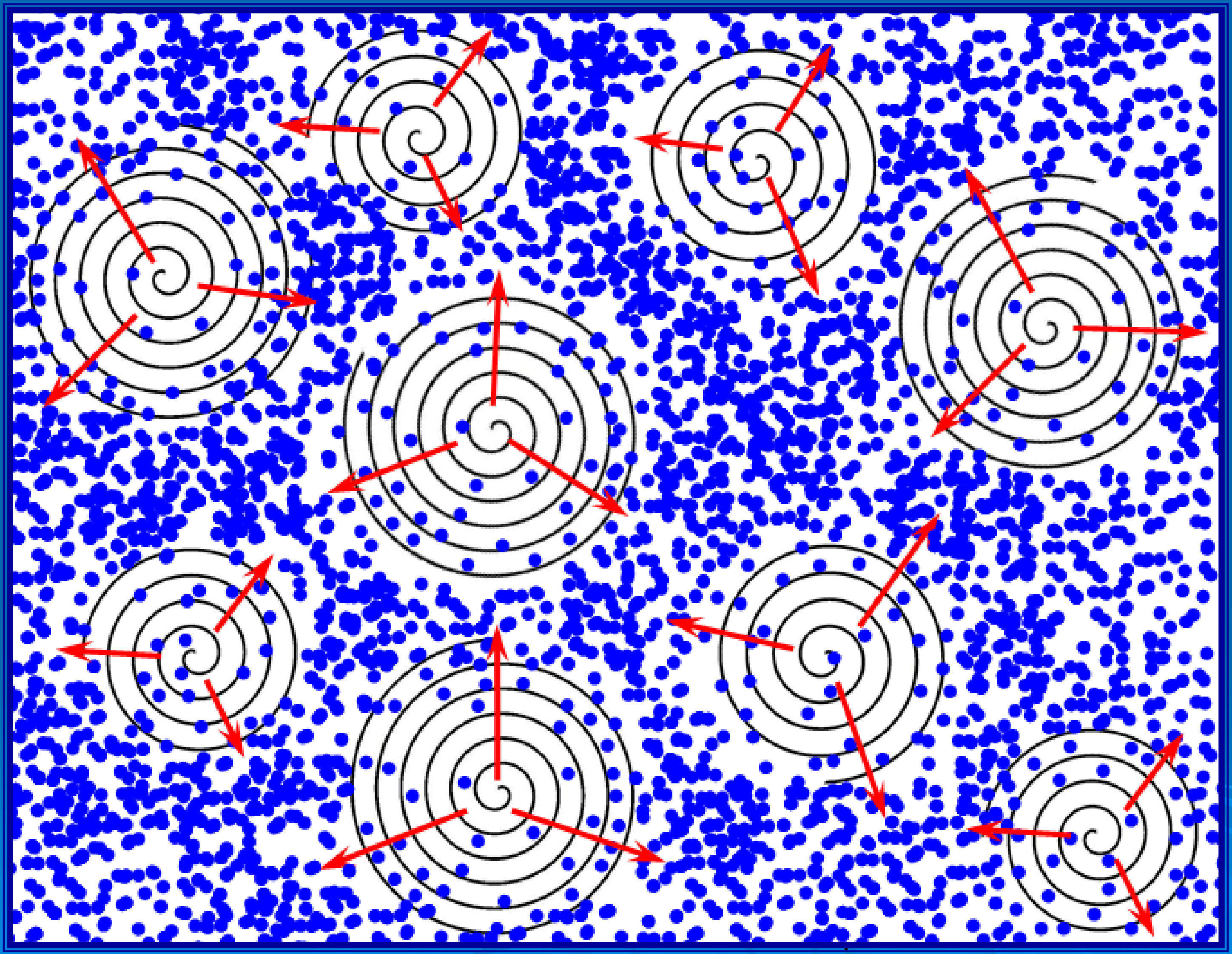
$$\mathrm{V_{eff}} = -\frac{\tau_s P}{\rho T} \langle \tau\, \mathbf{v}(\mathbf{x})\, \nabla^2 \theta(\mathbf{x}) \rangle = \frac{\tau_s P}{\rho T} \int \tau(k)\, k^2 \langle \mathbf{u}(\mathbf{k})\, \theta(-\mathbf{k}) \rangle\, d\mathbf{k}$$

Turbulent Thermal Diffusion

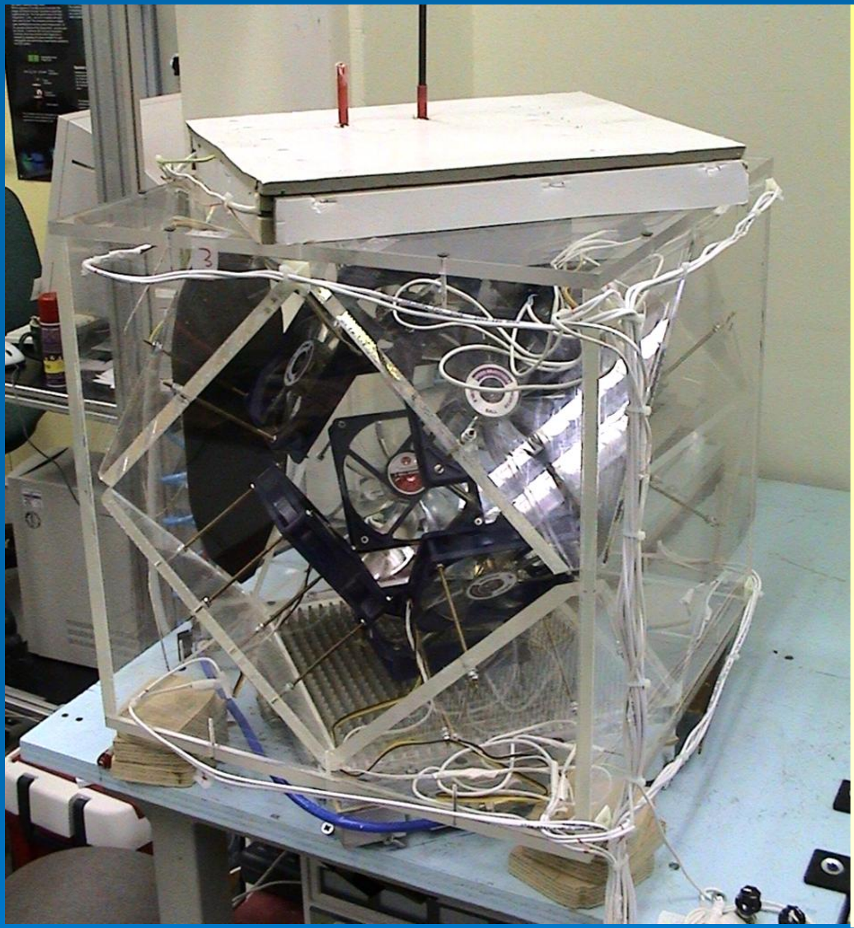


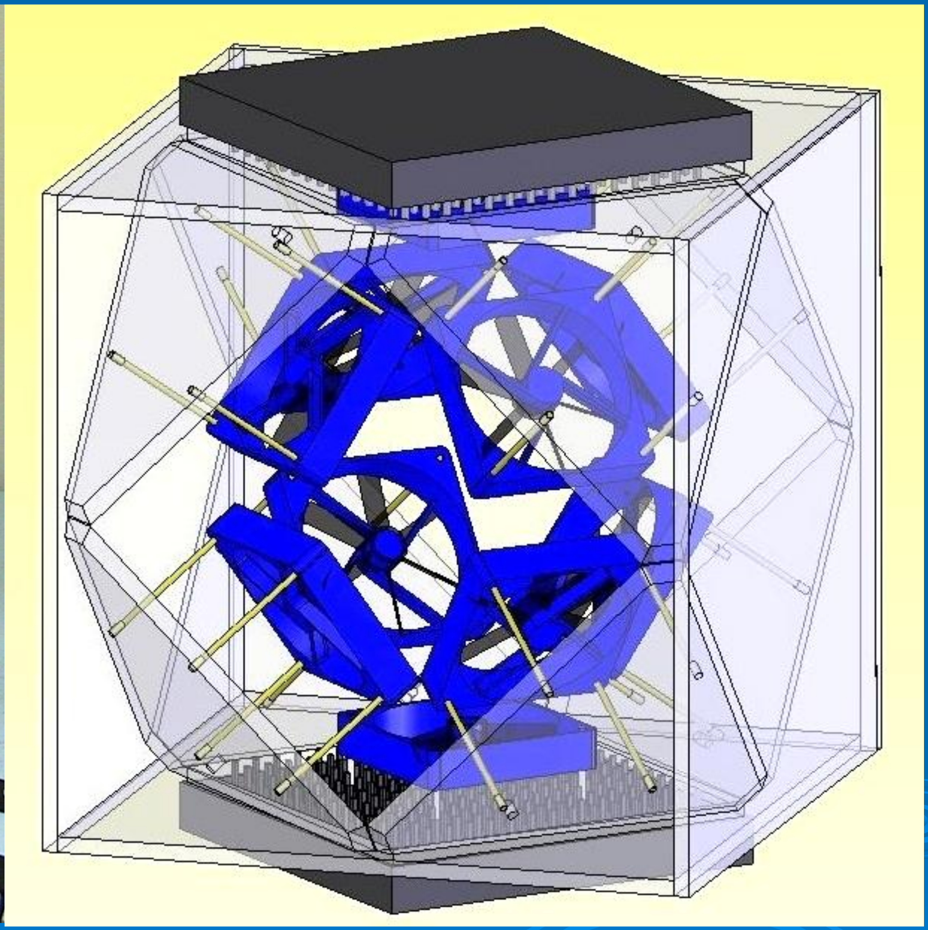
Non-diffusive mean flux of particles is in the direction of the mean heat flux (i.e., in the direction of minimum fluid temperature).

Particle Inertia Effect



Experimental set–up with ten fans





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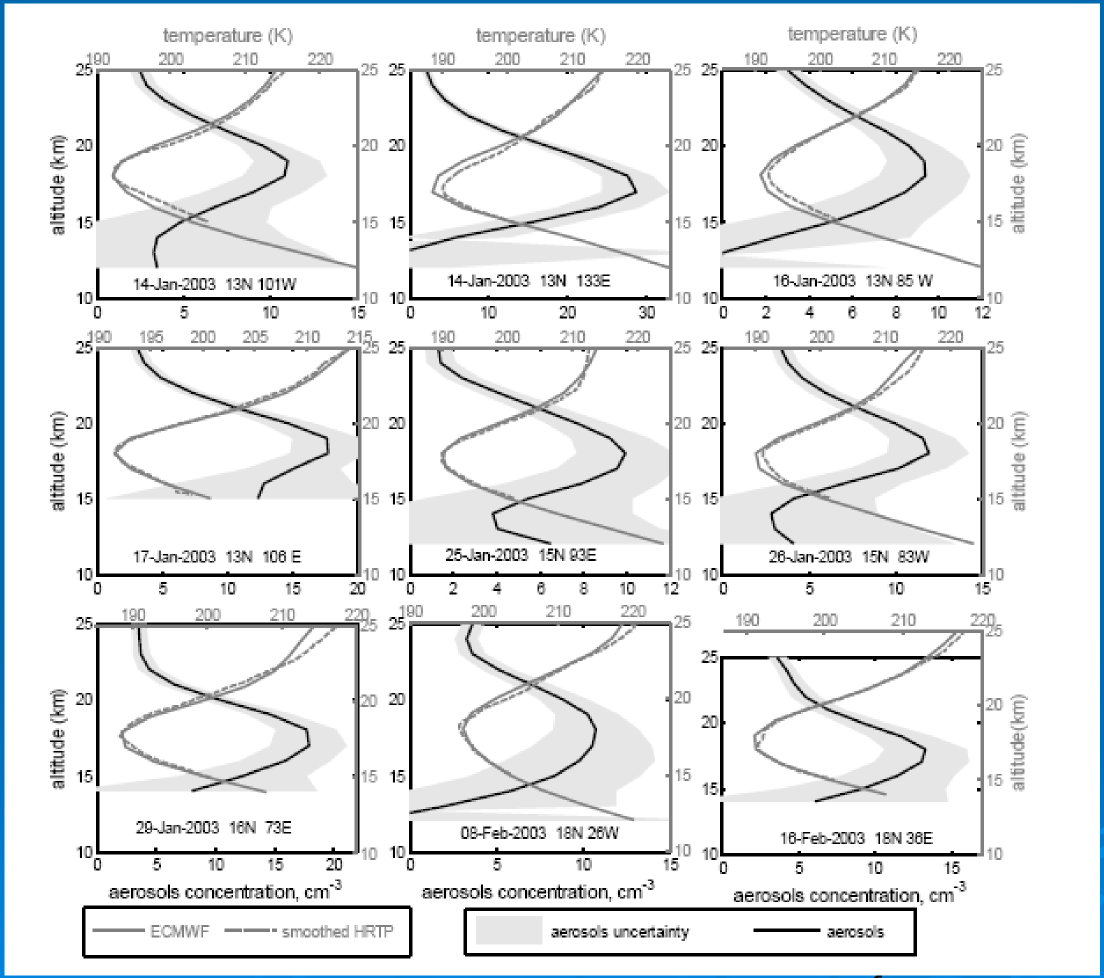
Distribution of Number Density of Aerosols (black) and Mean Temperature Distribution (gray) (Satellite Gomos Data)



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Particle Fluctuations in Turbulent Flow

- **Instantaneous particle number density:**

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\,\mathbf{v}) - D_m\,\Delta n = 0$$

- **Mean particle number density:** $n = N + n', \qquad N = \langle n \rangle$

$$\frac{\partial N}{\partial t} + \boldsymbol{\nabla} \cdot \langle n\,\mathbf{v} \rangle - D_m\,\Delta N = 0$$

$$\boldsymbol{\nabla} T \neq 0$$

$$\boldsymbol{\nabla} N \neq 0$$

- **Fluctuations of particle number density:**

$$\frac{\partial n'}{\partial t} + \boldsymbol{\nabla} \cdot (n'\,\mathbf{v} - \langle n'\,\mathbf{v} \rangle) - D_m\,\Delta n' = -N\,(\boldsymbol{\nabla} \cdot \mathbf{v}) - (\mathbf{v} \cdot \boldsymbol{\nabla})N$$

- **Source of fluctuations:**

$$I_{n'} = -N\,(\boldsymbol{\nabla} \cdot \mathbf{v}) - (\mathbf{v} \cdot \boldsymbol{\nabla})N$$

Two-point Correlation Function

$$\nabla T \neq 0$$
$$N = \langle n \rangle$$

- **Fluctuations of particle number density:**

$$\frac{\partial n'}{\partial t} + \nabla \cdot (n' \mathbf{v} - \langle n' \mathbf{v} \rangle) - D_m \Delta n' = -N(\nabla \cdot \mathbf{v}) - (\mathbf{v} \cdot \nabla) N$$

- **Two-point correlation function：** $\Phi(t, \mathbf{R}) = \langle n'(t, \mathbf{x})\, n'(t, \mathbf{y}) \rangle$

$$\frac{\partial \Phi}{\partial t} = [B(\mathbf{R}) + 2\mathbf{U}^{(A)}(\mathbf{R}) \cdot \nabla + D_{ij}(\mathbf{R}) \nabla_i \nabla_j]\, \Phi(t, \mathbf{R}) + I(\mathbf{R})$$

- **Source of tangling clustering:** $\mathbf{U}^{(S,A)}(\mathbf{R}) = (1/2)\, [\mathbf{U}(\mathbf{R}) \pm \mathbf{U}(-\mathbf{R})]$

$$I(\mathbf{R}) = B(\mathbf{R}) N^2 + \mathbf{U}^{(S)}(\mathbf{R}) \cdot \nabla N^2 + D^T_{ij}(\mathbf{R})\, (\nabla_i N)\, (\nabla_j N)$$

where $D_{ij} = 2 D_m \delta_{ij} + D^T_{ij}(0) - D^T_{ij}(\mathbf{R})$

$$B(\mathbf{R}) \approx 2 \langle \tau\, (\nabla \cdot \mathbf{v}(\mathbf{x}))\, (\nabla \cdot \mathbf{v}(\mathbf{y})) \rangle$$

$$\mathbf{U}_i(\mathbf{R}) \approx -2 \langle \tau\, \mathbf{v}_i(\mathbf{x})\, (\nabla \cdot \mathbf{v}(\mathbf{y})) \rangle$$

$$D^T_{ij}(\mathbf{R}) \approx 2 \langle \tau\, \mathbf{v}_i(\mathbf{x})\, \mathbf{v}_j(\mathbf{y}) \rangle$$

Source of Tangling Clustering $\boldsymbol{\nabla} T \neq 0$

- **Fluctuations of particle number density:** $N = \langle n \rangle$

$$\frac{\partial n'}{\partial t} + \boldsymbol{\nabla} \cdot (n' \mathbf{v} - \langle n' \mathbf{v} \rangle) - D_m \Delta n' = -N (\boldsymbol{\nabla} \cdot \mathbf{v}) - (\mathbf{v} \cdot \boldsymbol{\nabla}) N$$

- **Source of fluctuations:**

$$B(\mathbf{R}) \approx 2 \langle \tau [\boldsymbol{\nabla} \cdot \mathbf{v}(\mathbf{x})] [\boldsymbol{\nabla} \cdot \mathbf{v}(\mathbf{y})] \rangle = \frac{2 \tau_s^2}{\rho^2} \langle \tau [\nabla^2 p(\mathbf{x})] [\nabla^2 p(\mathbf{y})] \rangle$$

$$B(\mathbf{R}) = \frac{2 \tau_s^2 P^2}{\rho^2 T^2} \int \tau(k) \, k^4 \langle \theta(\mathbf{k}) \, \theta(-\mathbf{k}) \rangle \, e^{i \mathbf{k} \cdot \mathbf{R}} \, d\mathbf{k}$$

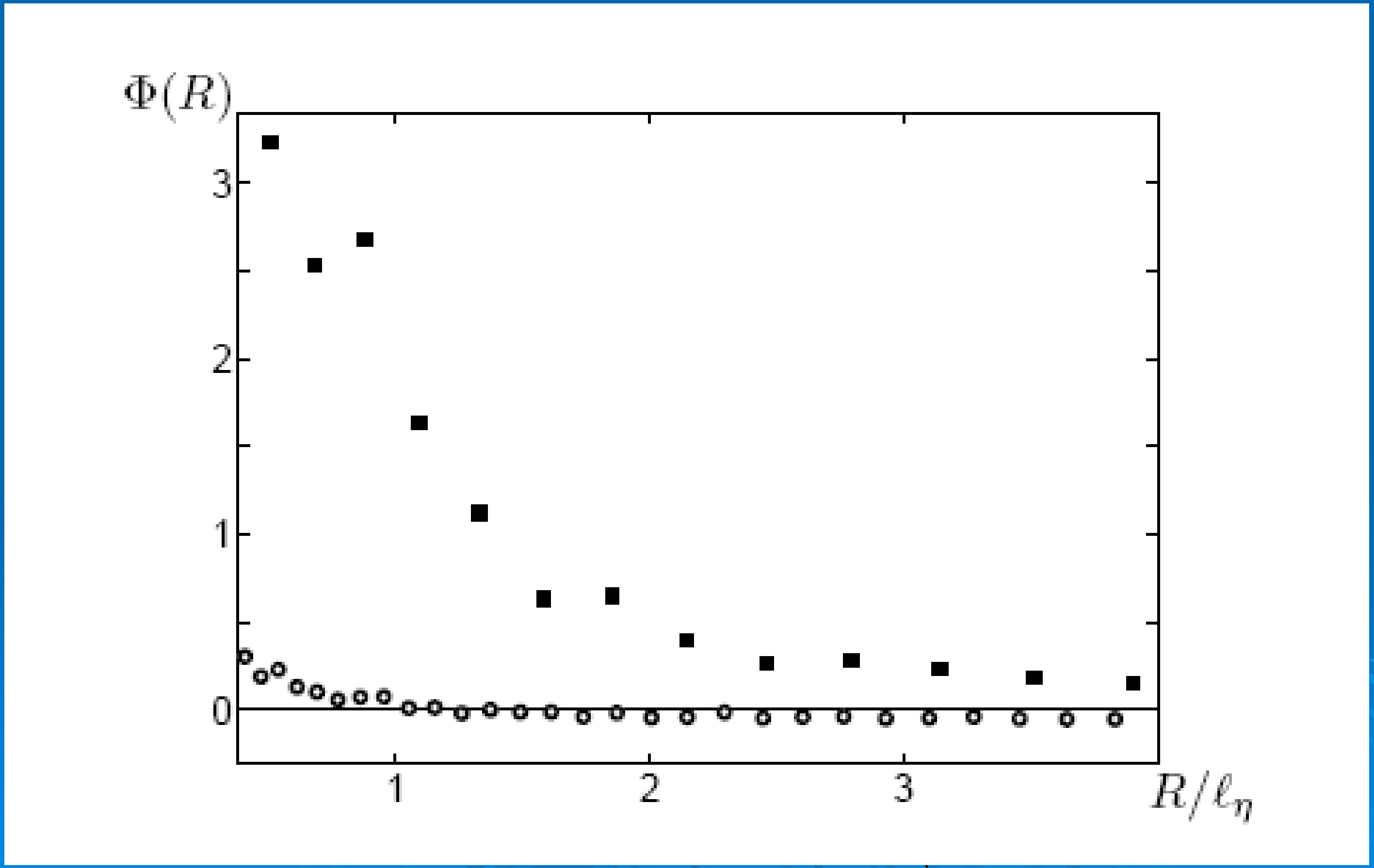
- **Tangling temperature fluctuations:** $\frac{\partial \theta}{\partial t} \propto -(\mathbf{v} \cdot \boldsymbol{\nabla}) T$

$$\langle \theta(\mathbf{k}) \, \theta(-\mathbf{k}) \rangle = 2 \tau^2(k) \langle u_i(\mathbf{k}) \, u_j(-\mathbf{k}) \rangle \, (\nabla_i T) \, (\nabla_j T)$$

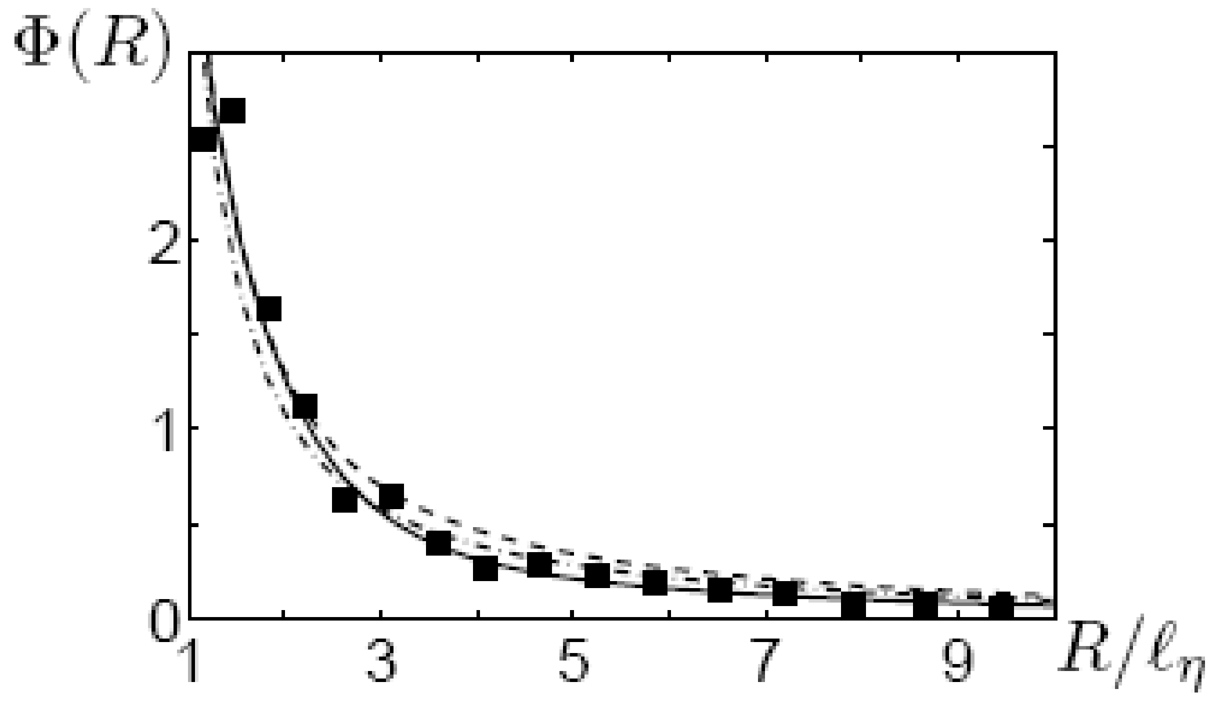
- **Source of tangling clustering of inertial particles:**

$$B(\mathbf{R}) = -\frac{8 \, \tau_s^2 \, k_b^2}{m_\mu^2 \, u_0^2} (\nabla_z T)^2 \, \tau_0 \, \Delta \langle u_z(\mathbf{x}) \, u_z(\mathbf{y}) \rangle$$

Normalized second-order correlation function determined in our experiments for (i) inertial clustering (isothermal turbulence, circles) (ii) tangling clustering (non-isothermal turbulence, squares)



Normalized second-order correlation function determined in our experiments (filled squares) and from our theoretical model (solid line)



References (Small-scale Effects)

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Conclusions

Small-scale effects

- We have predicted theoretically and detected in laboratory experiments **a new type of particle clustering** (namely, **tangling clustering of inertial particles**) in a stably stratified turbulence with imposed mean vertical temperature gradient.

- We have demonstrated that in the laboratory stratified turbulence the **tangling clustering** is much **more effective** than a **pure inertial clustering** (preferential concentration) that has been observed in isothermal turbulence, Re=250; $d_p = 10\,\mu$m.

- In the experiments **the correlation function for the inertial clustering** in isothermal turbulence is **much smaller** than that for the **tangling clustering** in non-isothermal turbulence.

- Our theoretical predictions are in a good agreement with the obtained experimental results.

THE END